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LEONTIEF SEMINAR

1. Clark — Anzio-Knight

Relation of probs to decisions

Decisions function of (relative desirability; relative likelihood)

Hold r.d constant; find r.l.

ROULETTE: placing wager

1. Ramsey-Savage notion of "more likely than": $a \alpha b > a \beta b$

α	$\bar{\alpha}$
a	b
e	c

2. $a \alpha b$ (i) $a \bar{\alpha} b \Rightarrow \alpha = \bar{\alpha} = \frac{1}{2}$

	α	$\bar{\alpha}$
I	a	b
II	b	a

MINIMAX

5. To get numerical utilities, use $\alpha = \frac{1}{2}$; for given a, b, find $c \ni a \alpha b = c$

Then $\frac{1}{2} u(a) + \frac{1}{2} u(b) = u(c)$, or $u(a) - u(c) = u(c) - u(b)$

4. With these utilities, find other probs: $x = y \beta z \Rightarrow \beta = \dfrac{u(x) - u(z)}{u(z) - u(y)}$

e.g.

	β	$\bar{\beta}$
I	9	0
II	3	3

I (i) II $\Rightarrow \beta = \frac{1}{3}$

5. Or, without measuring utilities, find other probs:

α	β	$\overline{\alpha\cup\beta}$
a	b	b
b	a	b
b	b	a

$I = II = III \Rightarrow \alpha = \beta = \overline{\alpha\cup\beta} = \frac{1}{3}$

If these events have "objective" probs, they may be different; we aren't interested in them.

Discuss MINIMAX, Game Against Nature

6. To get probs & utils, Savage Postulates on choices among gambles:

e.g. P. 1 — Simple ordering:

a	b
c	d
e	f

P2: Sure-thing Principle

a	b	c
b	a	c
a	b	d
b	a	d

7. ~~Qu~~ Do people obey these postulates? If they do, they act *as if* they assigned *numerical* probs (and these probs can be *measured* by their actions.) Normative or descriptive?

8. Weaker ~~Q~~ question: do they act as if they assigned *qualitative probs*?

Properties: ~~i) $a\alpha b > a\beta b \Leftrightarrow b\beta a > b\alpha a$~~ (i.e. $a\bar{\beta}b > a\bar{\alpha}b$)

1) $\alpha > \beta \Leftrightarrow \bar{\beta} > \bar{\alpha}$

2) $\alpha = \beta \Leftrightarrow \bar{\alpha} = \bar{\beta}$

3) $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta} \Leftrightarrow \alpha = \beta$

4) $\alpha > \beta \Leftrightarrow \alpha \cup \gamma > \beta \cup \gamma$

Notions of additivity, probs adding to *constant*.

9: These correspond to: 1) $a\alpha b > a\beta b \Leftrightarrow a\bar{\beta}b > a\bar{\alpha}b$

2) $=$ $=$

3) $a\alpha b = a\bar{\alpha}b,\ a\beta b = a\bar{\beta}b \Leftrightarrow a\alpha b = a\beta b$

4) $a\alpha b > a\beta b \Leftrightarrow a(\alpha \cup \gamma)b > a(\beta \cup \gamma)b$

1. =

2. =

3. ±

4. =

5. ~~8~~ 5

6. ~~8~~ 5

7. 0

8. 0

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10. Person who violates these isn't acting as though he assigned <u>probs</u> at all; at least, there must be other influences on him. Can't infer a <u>probability</u> relation over events for him.

~~II~~ Exp: I

[illegible]

I

? R_1, ? B_1

II

50 R_2, 50 B_2

I. How much would you pay for: \$100 R_2 0 ?

for 100 R_1 0 ?

for 100 B_1 0 ?

for 100 B_2 0 ?

a b
c b

a b (c d)
f g (d e)

e b (x y)
f g (m n)

5.

1. 2. Elicit answers: which would you prefer: 100 R_2 0 or 100 R_1 0 ?

100 B_2 0 or 100 B_1 0 ?

100 R_1 0 or 100 B_1 0 ?

100 R_2 0 or 100 B_2 0 ?

3. Note: $R_1 = B_1$ and $R_2 = B_2$ but $R_1 \neq R_2$!

$R_1 < R_2$ <u>and</u> $B_1 < B_2$!

4. Variations: U_I sample: 1R, 1B U_{II} 50, 50

U_I ?R, ?B but add 1 R U_{II} 50, 50

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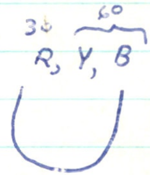
8. Actions aren't a function, then, of payoffs (constant) and probabilities alone.

~~7~~

12. EXP. 2



	R	Y	B
I	a	b	b
II	b	a	b
III	a	b	~~b~~ a
IV	b	a	a

Note: Sure-thing Post. violated
R > Y, but R ∪ B < Y ∪ B

a	b
c	b
d	b
e	b

$>$

$A > B \Leftrightarrow \bar{A} < \bar{B}$

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13. What is going on?

Ambiguity. Minimax w.r.t. ambiguity.

30	60
a	b
b	b
a	b
b	a

Changing const. column affects ambiguity.

14. On maximax. Wishful.

15. On Hurwicz. Probs depend on payoffs, given ambiguity.

16. Importance: U_I 5R, 4B U_{II} 49R, 51B $1 R_2 0$ or $1 R_1 0$?

$40 R_2 - 30$ or $40 R_1 - 30$

The minimax of regret (which seems more "sensible") obeys P2; hence gives "consistent" answers here; actual answers I get, rule out minimax regret as basis of their actual decisions.

Minimax regret w.r.t. ambiguous events
(don't suppress the fact that outcomes are associated in columns
Thus, weights will depend not only on events ~~or~~ payoffs, but on set of actions considered

	0	a	a
	a	0	a
	0	a	0
→	a	0	0

	0	a	0	0	a
→	a	0	0	a	0
	0	a	0	0	a
→	a	0	0	a	0

a	b	b
b	a	b
a	b	a
b	a	a

1	0	0
1	0	0
1	0	0
1	0	0

Chipman's Exp. and Savage Axioms

S: P4 (p. 31)

	$\bar{B}$ / A	$\bar{A}$ / B
f_A	f	f'
f_B	f'	f

$f > f'$

$g > g'$

g_A	g	g'
g_B	g'	g

P4: $f_A \leq f_B \iff g_A \leq g_B$

$\stackrel{\text{def}}{\equiv} A \leq B$

This corresponds to Luce's axiom of independence of subj. prob. and payoffs:

	A	B
I	a	b
II	b	a

$a > b$

$A \geq B \iff$ I p II for any a, b; $a > b$.

This will be violated by (among others) a man who shifts to minimax (maximax) at high levels of payoff (with A, B ambiguous). He will shift from preference to indifference. Man who is equally pessimistic at all levels will be indifferent at all levels.

over

S p. 32: 3. If $B \leq C$, then $\sim C \leq \sim B$. But if C is ambiguous, B not, then man, at given payoff level, may minimax for C and $\bar{C}$, max for B; he may have $B > C$ and $\bar{B} > \bar{C}$. Chipman's case if $p = \frac{1}{2}$ and $s = \frac{1}{2}$.

p. 33, 6: If $B = \bar{B}$ and $C = \bar{C}$ then $B = C$. see above.